Testing the expectations hypothesis when interest rates are near integrated

Meredith Beecheya, Erik Hjalmarssonb,*, Pär Österholmc

a Division of Monetary Affairs, Board of Governors of the Federal Reserve System, 20th and C Streets, Washington, DC 20551, USA
b Division of International Finance, Board of Governors of the Federal Reserve System, 20th and C Streets, Washington, DC 20551, USA
c Department of Economics, Uppsala University, Box 513, 751 20 Uppsala, Sweden

Article info

Article history:
Received 11 June 2008
Accepted 15 October 2008
Available online 1 November 2008

JEL classification:
C22
G12

Keywords:
Bonferroni tests
Cointegration
Expectations hypothesis
Near integration
Term premium

Abstract

Nominal interest rates are unlikely to be generated by unit-root processes. Using data on short and long interest rates from eight developed and six emerging economies, we test the expectations hypothesis using cointegration methods under the assumption that interest rates are near integrated. If the null hypothesis of no cointegration is rejected, we then test whether the estimated cointegrating vector is consistent with that suggested by the expectations hypothesis. The results show support for cointegration in 10 of the 14 countries we consider, and the cointegrating vector is similar across countries. However, the parameters differ from those suggested by theory. We relate our findings to existing literature on the failure of the expectations hypothesis and to the role of term premia.

Published by Elsevier B.V.

1. Introduction

Empirical tests of the expectations hypothesis of the term structure often fail to find support for the theory. The logic underlying the theory, that expectations of future short interest rates shape the term structure of longer interest rates, is intuitive, appealing, and a common assumption in macroeconomic modelling. However, the predictability of excess returns shown by Fama and Bliss (1987), Campbell and Shiller (1991) and more recently by Cochrane and Piazzesi (2005) undermines the premise that long interest rates are rational expectations of future short rates up to a constant term premium. Rather, such evidence points strongly toward time-varying risk premia. Indeed, Dai and Singleton (2002) demonstrate that interest rates adjusted for time-varying risk premia estimated from dynamic term structure models meet the predictions of the expectations hypothesis in traditional excess-return regressions.

One strand of the empirical literature on interest rates has sought to test the expectations hypothesis using the techniques of cointegration. As pointed out by Engle and Granger (1987) in their seminal paper on cointegration, if nominal interest rates are generated by a unit-root process, cointegration between yields of different maturities is a necessary condition for the validity of the expectations hypothesis. Intuitively, if interest rates are integrated of order one, the expectations hypothesis implies that the spread between any pair of yields is stationary. Following Engle and Granger’s early work, several studies have taken a similar path and have found only mixed evidence for the expectations hypothesis; see, for example, Campbell and Shiller (1987), Boothe (1991), Hall et al. (1992), Zhang (1993) and Lardic and Mignon (2004).

It is an empirical fact that nominal interest rates are highly persistent and the poor power of traditional univariate Dickey–Fuller type tests against the null of a unit root (Stock, 1994) has led many researchers to conclude that interest rates are integrated of order one. Moreover, the convenience of working with established results for integrated processes has made it attractive to assume the presence of a unit root for empirical purposes. As such, nominal interest rates have been treated as integrated of order one in numerous empirical papers, including Karfakis and Moschos (1990), Bremnes et al. (2001), Chong et al. (2006), Kleimeier and Sander (2006), De Graeve et al. (2007) and Liu et al. (2008). For theoretical purposes, the unit-root assumption can also be a useful modelling device to capture stylistically the highly persistent nature of interest rates in finite samples, exemplified by Cogley and Sargento’s (2001) approach to modelling the dynamics of a system of macroeconomic variables.

However, the exact unit-root assumption for nominal interest rates can be questioned on both empirical and theoretical grounds.
Empirically, standard tests for unit roots have difficulty discriminating true integration from highly-persistent dynamics but panel unit-root tests – which tend to be more powerful than univariate tests – tend to find support for mean reversion in nominal interest rates (Wu and Chen, 2001). Theoretically, it might be unsatisfactory to model interest rates as unbounded in the limit. More importantly though, most economic models predict that real interest rates possess a long-run equilibrium value, determined by the long-run rate of potential output growth and population growth, and agents’ rate of time preference and risk aversion. Similarly, consumption growth is typically viewed as stationary – albeit slowly mean reverting (Bansal and Yaron, 2004). The standard consumption Euler equation thus implies stationary real interest rates. Nominal interest rates have varied substantially in recent decades, partly reflecting the undulations of inflation expectations, but over the long-run, have wandered within reasonable bounds. Indeed, from an historical perspective, short-term nominal interest rates were in the range of 4–8% during the years of the Roman Empire, and western European commercial and mortgage borrowing rates moved in the 4–8% range from the 13th to 17th century (Homer and Sylla, 1996). Private nominal interest rates are in a similar range today, an outcome that would be virtually impossible if interest rates possessed a unit root but consistent with a highly-persistent data generating process.

Unfortunately, standard cointegration-based inference designed for unit-root data is typically not robust to even small deviations from the unit-root assumption. Results will generally be biased when the autoregressive roots in the data are close, but not identical, to unity; this is true both for actual tests of the cointegrating rank (Hjalmarsson and Österholm, 2007a,b) as well as for inference on the cointegrating vector (Elliott, 1998). Conclusions of empirical tests of the expectations hypothesis that rely on traditional cointegration are therefore called into question. As such, empirical analysis within a framework that acknowledges the high persistence in nominal interest rates but that does not impose the strict assumption of a unit root is desirable but very few examples exist in the literature. To our knowledge, exceptions include one study of the Fisher effect (Lanne, 2001) and one on the term structure of interest rates (Lanne, 2000).

In this paper, we revisit the question of cointegration between yields of different maturities using methods that allow for valid inference when data are near integrated. That is, we assume that interest rates are highly persistent with autoregressive roots that are close to unity. The empirical framework nests the standard unit-root assumption used in the traditional cointegration studies mentioned above, but also permits interest rates to be (slowly) mean reverting. We test for cointegration using the recently developed methods of Hjalmarsson and Österholm (2007a) that are robust to deviations from the pure unit-root assumption. Tests of the term structure of interest rates in several developed and emerging economies, namely Australia, Canada, Hungary, India, Japan, Mexico, New Zealand, Poland, Singapore, South Africa, Sweden, Switzerland, the United Kingdom and the United States. The results provide strong support for cointegration between long and short interest rates in 10 of these countries. Among the developed countries, results show support for cointegration in Australia, Canada, New Zealand, Sweden, Switzerland and the United States. Among the emerging economies, a similar result is returned for Hungary, Mexico, Poland and Singapore.

Cointegration is only one of two necessary conditions for the validity of the expectations hypothesis; the theory also contains strong predictions about the parameters of the cointegrating vector. However, earlier work has largely overlooked the interpretation of the parameters of the cointegrating vector. We test whether the parameters are consistent with the theoretically-suggested values using an extension of fully-modified estimation that is again robust to deviations from the pure unit-root assumption. Fully modified estimation of cointegrated systems was initially developed by Phillips and Hansen (1990) and discussed by Hjalmarsson (2007) in the context of predictive regressions with near unit-root variables; in the current paper, we extend those ideas further to accommodate inference in a general bivariate cointegrating relationship with nearly integrated variables. For the 10 countries in which cointegration is detected, the cointegrating vector does not coincide with that suggested by theory. In each case, long rates move by less than short rates, with the similarity of the estimated vectors across countries hinting at a common explanation. Indeed, the estimated cointegrating vectors suggest that another near-integrated variable that covaries inversely with the short rate also affect the term structure of longer interest rates. This is consistent with time-varying bond risk premia put forward by Campbell and Shiller (1991) and Dai and Singleton (2002) to explain the puzzling patterns of coefficients from yield-spread regressions, and with the countercyclical pattern of excess returns noted by Fama and Bliss (1987) and Cochrane and Piazzesi (2005). We conjecture that the phenomenon underlying the rejection of the expectations hypothesis in excess-return regressions may also be responsible for the failure of the expectations hypothesis in cointegration methods.

The remainder of the paper is organized as follows: Section 2 presents the theoretical framework regarding the term structure of interest rates and the econometric methodology. In Section 3, the empirical analysis is conducted and the results discussed and Section 4 concludes. Some sensitivity analysis is presented in the appendix.

2 Theoretical framework

This section presents the theoretical motivation for our empirical tests. We briefly lay out the expectations hypothesis of the term structure data move on to the econometric methodology of testing for cointegration between near-integrated processes.

2.1. The term structure of interest rates

We begin with a statement of the expectations hypothesis of the term structure similar to that found in Campbell and Shiller (1991):

\[ \Delta l_t^n = \frac{1}{n} \sum_{j=0}^{n-1} E_t (\Delta l_{t+j}) + \xi^n. \]  

(1)

Simply put, the expectations hypothesis posits that the interest rate on a longer-term n-period bond, \( \Delta l_t^n \), is equal to the average of the expected path of future one-period interest rates over the life of the bond. For the 10 countries in which cointegration is detected, the cointegrating vector does not coincide with that suggested by theory. In each case, long rates move by less than short rates, with the similarity of the estimated vectors across countries hinting at a common explanation. Indeed, the estimated cointegrating vectors suggest that another near-integrated variable that covaries inversely with the short rate also affect the term structure of longer interest rates. This is consistent with time-varying bond risk premia put forward by Campbell and Shiller (1991) and Dai and Singleton (2002) to explain the puzzling patterns of coefficients from yield-spread regressions, and with the countercyclical pattern of excess returns noted by Fama and Bliss (1987) and Cochrane and Piazzesi (2005). We conjecture that the phenomenon underlying the rejection of the expectations hypothesis in excess-return regressions may also be responsible for the failure of the expectations hypothesis in cointegration methods.

The remainder of the paper is organized as follows: Section 2 presents the theoretical framework regarding the term structure of interest rates and the econometric methodology. In Section 3, the empirical analysis is conducted and the results discussed and Section 4 concludes. Some sensitivity analysis is presented in the appendix.

2. Theoretical framework

This section presents the theoretical motivation for our empirical tests. We briefly lay out the expectations hypothesis of the term structure then move on to the econometric methodology of testing for cointegration between near-integrated processes.

2.1. The term structure of interest rates

We begin with a statement of the expectations hypothesis of the term structure similar to that found in Campbell and Shiller (1991):

\[ \Delta l_t^n = \frac{1}{n} \sum_{j=0}^{n-1} E_t (\Delta l_{t+j}) + \xi^n. \]  

(1)

Simply put, the expectations hypothesis posits that the interest rate on a longer-term n-period bond, \( \Delta l_t^n \), is equal to the average of the expected path of future one-period interest rates over the life of the bond.
bond, \(i_{t-j}, j = 0, \ldots, n-1\), plus a constant term premium, \(c_t\), that may differ across maturities, \(n\). Expectations of future short interest rates are assumed to be rational.

Subtracting \(i_t\) from both sides of Eq. (1) and rearranging terms, the expectations hypothesis states that the yield spread between an \(n\)-period bond and the current one-period rate can be viewed as the weighted average of expected changes in short interest rates over the life of the bond:

\[
E^t_l - i_t = \frac{1}{n} \sum_{p=1}^{n-1} \sum_{j=1}^{p} E_t(\Delta i_{t-j}) + c_t^n.
\]  

(2)

Paired with the assumption that one-period nominal interest rates are integrated or near integrated, the sum of first differences on the right-hand side of Eq. (2) must be stationary. This in turn implies that any \(n\)-period yield should be cointegrated with the one-period interest rate and that the cointegrating vector will be \((1, -1)\). Note that the term premium in Eq. (2) has been generalised to include a time subscript, \(c_t^n\). This has been done deliberately, acknowledging that testing for cointegration between the left-hand-side variables cannot discriminate between theories of the term structure in which term premia are constant versus theories in which term premia are time-varying but stationary, as noted by Miron (1991) and Lanne (2000). As such, evidence for cointegration does not speak to the strict form of the expectations hypothesis.

2.2. Econometric methodology

The main starting point of our empirical analysis is the assumption that both the long and short interest rates follow near unit-root processes with local-to-unity roots \(\rho_1 = 1 + c_1/T\) and \(\rho_s = 1 + c_s/T\), respectively, where \(T\) is the sample size; there is no requirement that \(\rho_1 = \rho_s\). Thus, if \(l_t = (\tilde{i}_t, \tilde{c}_t)\), it is assumed that \(\tilde{l}_t = A\tilde{l}_{t-1} + \tilde{u}_t\),

\[
A = I + C/T \quad \text{and} \quad C = \text{diag}(c_1, c_s),
\]

where \(A\) is a \(2 \times 2\) matrix with \(A = \text{diag}(\rho_1, \rho_s)\), and \(C = \text{diag}(c_1, c_s)\). The innovations \(u_t\) are assumed to satisfy a general linear, or infinite moving average, process. Although not necessary for the formal econometric procedures that are used, we will maintain the assumption throughout the paper that \(c_1, c_s < 0\), which rules out explosive processes. Since we are analysing interest rates, this is clearly a weak assumption.

This specification of the data-generating process for the interest rates at different horizons is a generalisation of a pure unit-root model where interest rates at all maturities contain a unit root. Compared to a pure unit-root process, Eq. (3) offers considerably more flexibility. First, the model allows for deviations from the strict unit-root assumption while still preserving, also asymptotically, the empirically observed feature that interest rates are highly persistent. Second, the model above also allows for different degrees of persistence in interest rates with different maturities. Although it is natural to believe that interest rates at different horizons share the same persistence if the expectations hypothesis is true, this does not necessarily hold under an alternative hypothesis.

As shown in the empirical analysis below, the near unit-root model suggested here finds more support in the data than the pure unit-root model. For several series, the null of a unit root can be rejected and unbiased estimates of \(c_1\) and \(c_s\) are typically negative, suggesting some mean reversion in the data.

While the concept of cointegration readily carries over to variables with near unit roots, as discussed in Phillips (1988), traditional cointegration tests based on a unit-root assumption are biased (Hjalmarsson and Österholm, 2007a,b). Furthermore, if the presence of cointegration between near unit-root variables is established, subsequent tests on the cointegrating vector will be biased if performed under the assumption of unit roots in the data (Elliott, 1998). Moreover, even if null of a unit root in the variables cannot be rejected, there is no guarantee that the data contain an exact unit root, as unit-root tests have low power against alternatives close to a unit root. Thus, unless there are very strong a priori reasons to believe that there is an exact unit root in the data, based on some theoretical argument for instance, any cointegration test based on the unit-root assumption is subject to a potential bias.

In the next two sections, we outline methods for dealing with an unknown local deviation from the pure unit-root assumption, both for actual tests of cointegration as well as for inference on the cointegrating vector.

2.2.1. Robust tests for cointegration

We focus on residual-based tests of cointegration and rely on the methods proposed by Hjalmarsson and Österholm (2007a) to deal with the issues raised by near unit-root variables. In effect, a residual-based cointegration test evaluates whether the residuals from the empirical regression contain a unit root. However, if the original data are in fact near integrated, with a root less than unity, the test will over-reject as the residuals will not contain a unit root even if there is no cointegration.

The idea behind Hjalmarsson and Österholm’s test is therefore to replace the critical values of the test under the unit-root assumption with critical values based on a conservative estimate of the local-to-unity root in the original data. Intuitively, if one views a residual-based test of cointegration as a test of whether there is less persistence in the residuals than in the original data, this test is only valid if the persistence of the original data is not overstated.

In particular, we will use a modified version of the traditional Engle–Granger (EG) test for cointegration. Consider the cointegrating regression

\[
\tilde{i}_t = \kappa + \beta \tilde{c}_t + \tilde{v}_t,
\]

where the fitted residuals \(\tilde{v}_t\) are tested for a unit root according to

\[
\Delta \tilde{v}_t = \tau \tilde{v}_{t-1} + \sum_{i=1}^{p} \varphi_i \Delta \tilde{v}_{t-i} + w_t.
\]

The EG test is defined as the \(t\)-statistic on \(\tilde{v}_t\) from Eq. (5).

As shown by Hjalmarsson and Österholm (2007a), under the near unit-root assumption, the limiting distribution of the EG test statistic depends on the unknown matrix of local-to-unity parameters \(C = \text{diag}(c_1, c_s)\), and the critical values of the EG test are thus unknown in the near unit-root case. In order to obtain a practically feasible procedure, Hjalmarsson and Österholm first show that the
relevant critical values of this limiting distribution are primarily a
function of \(c_i\), the persistence of the “dependent” variable, and are
almost invariant to the value of \(c_i\), the persistence of the regressor
variable. They therefore suggest using critical values based on
\(C_1 = \text{diag}(c_1, c_i)\), rather than \(C = \text{diag}(c, c_i)\), which greatly simplifies
the feasible implementation.

Furthermore, the critical values for the EG test are increasing in \(c_i\). Therefore, in order to form a correctly sized test, all that is
needed is a “sufficiently” conservative estimate of \(c_i\). As shown
by Stock (1991), a confidence interval for \(c_i\) can be obtained by
inverting a unit-root test-statistic for \(i^*_t\); since only the lower bound
matters here, a one-sided confidence interval is appropriate.

According to Bonferroni’s inequality, if the confidence level of this
one-sided confidence interval is 95% and the nominal size of the EG
test, evaluated using the critical values based on the lower bound
for \(c_i\), is 5%, then the actual size of the cointegration test is less than
or equal to 10%.

As shown by Hjalmarsson and Österholm, however, Bonferroni’s
inequality tends to be strict and the actual size of the test procedure
just described is, in fact, very close to zero. Instead, they find
that if the EG test is evaluated at the 5% level and the critical values
are based on the lower bound of a one-sided confidence interval for \(c_i\) with confidence level of 50%, the overall test of
cointegration will have an actual size of approximately 5%. Thus,
the median unbiased estimate of \(c_i\) can be used to form the critical
values.

In summary, the following procedure to test the null of no coin-
tegration for near-integrated variables will thus be used in the
analysis here:

(i) Obtain the value of the test statistic from a standard imple-
mentation of the EG test.

(ii) Invert the test statistic from the Dickey–Fuller test with GLS
detrending (ADF–GLS) of Elliott et al. (1996) and obtain the
median unbiased estimate of \(c_i\); denote this estimate \(\hat{c}_i\).

For a given value of the ADF–GLS test statistic, the corre-
sponding median unbiased estimate can be found in Table

(iii) Compare the EG test statistic to the critical value in Table A3
in Hjalmarsson and Österholm (2007a), corresponding to the
distribution under \(c_i\) if \(c_i < 0\). If \(c_i > 0\), use the critical values
from the traditional EG test, that is, for \(c_i = 0\).

We will refer to the test of cointegration constructed in the above
manner as the Bonferroni EG test.

2.2.2. Tests of restrictions on the cointegrating vector

If one does establish that variables are cointegrated, it is often of
interest to perform inference on the cointegrating vector. In the
current application, we are interested in whether the cointegrating
vector is \((1, -1)\), since this is an additional condition for the expec-
tations hypothesis to hold once cointegration between the short
and long interest rates has been established.

In the case with unit-root regressors, inference on the cointe-
grating vector is typically performed using standard tests based
on the estimates from some efficient estimation procedure of the
cointegrating vector, such as the dynamic OLS of Saikkonen
(1991) and Stock and Watson (1993) or the fully modified OLS
(FM-OLS) of Phillips and Hansen (1990). Using these efficient esti-
mation methods, which are asymptotically equivalent, the result-
ing test statistics have standard distributions. However, as shown
by Elliott (1998), this is no longer true when the data are nearly
integrated, and tests based on the unit-root assumption can be
highly misleading.

In this paper, we therefore apply an extension of the FM-OLS
procedure for nearly integrated regressors. The idea was developed
by Hjalmarsson (2007) for predictive regressions with nearly inte-
grated regressors. Given the predictive nature of that model, the
FM-OLS estimator needs to be slightly modified to fit the standard
(contemporaneous) cointegration regression studied here. The fol-
lowing results are derived under the assumption of cointegration,
such that Eq. (4) represents a true relationship, with a stationary
error term \(\nu_t\).

To fix notation, let \(u_t^*\) be the innovations to the short interest
rate variable; that is,

\[
\tilde{u}_t = \rho \tilde{u}_{t-1} + u_t^*,
\]

where \(\rho = 1 + c_i/T\). The model is now given by Eqs. (4) and (6). De-
note the joint innovations \(w_t = (\tilde{v}_t, u_t^*)^\top\) and suppose that \(w_t\) satisfy
a functional law such that \(T^{-1/2} \sum_{t=1}^{T} w_t = B(r) = BM(\Omega)(r)\),
where \(B(\cdot) = (B_1(\cdot), B_2(\cdot))^\top\) denotes a two-dimensional Brownian motion
with variance–covariance matrix \(Q = [(\Omega_{11}, \Omega_{12}), (\Omega_{12}, \Omega_{22})]^\top\); that
is, \(\Omega\), the long-run two-sided variance–covariance matrix for \(w_t\).

Further, let \(A_{12} = \sum_{t=1}^{T} \sum_{t} E[v_t u_t^*]\) and \(A_{22} = \sum_{t=1}^{T} v_t \tilde{v}_t\)
be the one-sided long-run covariance and variance, respectively.

In general, the OLS estimator of \(\beta\) in Eq. (4) is not efficient, and
the resulting test statistics have non-standard asymptotic distribu-
tions whenever there is a non-zero correlation between \(v_t\) and \(u_t^*\).

In the pure unit-root case, Phillips and Hansen (1990) therefore
suggest that the OLS estimator be “fully modified”. As shown in
Hjalmarsson (2007), in the near unit-root case, a similar method
can also be considered. However, the modification makes use of
the innovations \(u_t^*\), which can only be obtained by knowledge of
\(c_i\), which is unknown.\(^{4}\) We therefore first derive the estimator under
the assumption that \(c_i\) is known, and then discuss feasible methods
to deal with an unknown \(c_i\). Define

\[
\Delta_i \tilde{u}_t = \tilde{u}_t - \tilde{u}_{t-1} - \frac{c_i}{T} \tilde{u}_{t-1} = u_t^*,
\]

and let \(\tilde{L}_t = L_t - \Omega_{12} \tilde{v}_t \Delta_i \tilde{u}_t\) and \(A_{12}^* = A_{12} - \Omega_{12} \tilde{v}_t \tilde{v}_t^\top A_{22}\),
where \(\Omega_{12}, \Omega_{22}, A_{12}\) and \(A_{22}\) are consistent estimates of the respective
parameters and \(\tilde{L}_t\) and \(\tilde{L}_t^\top\) denote the demeaned data. The fully modified
OLS estimator is given by

\[
\hat{\beta}^* = \left( \sum_{t=1}^{T} \tilde{L}_t^\top \tilde{L}_t \right)^{-1} \left( \sum_{t=1}^{T} \tilde{L}_t^\top \tilde{L}_t \right)^{-1} \hat{\beta}.
\]

Define \(\omega_{11,2} = \omega_{11} - \Omega_{12} \tilde{v}_t \tilde{v}_t^\top \omega_{22}\) and as shown in Hjalmarsson
(2007), as \(T \rightarrow \infty\),

\[
T \left( \hat{\beta}^* - \beta \right) = \text{MN} \left( 0, \omega_{11,2} \right) \left( 1 \int_0^T \tilde{L}_t^\top \tilde{L}_t \right)^{-1}.
\]

\(^{4}\) Note that it is possible to estimate \(\rho\) consistently, but not precisely enough to
consistently identify \(c_i\) or \(c_i < 1\); median unbiased estimates can also be obtained,
as presented in the empirical section, but these are again not consistent estimates.
Using an estimate of \(\rho\) to obtain estimates of the innovations \(u_t^*\) in the FM-OLS
procedure discussed here will lead to biased tests.

\(^{5}\) Since \(c_i\) is consistently estimable, it follows that \(\Omega, A_{12}\), and \(A_{22}\) are consistently
estimable, even though \(c_i\) is not.
are obtained, the FM-OLS procedure for near unit-root variables is identical to the standard one used in the unit-root case.

For a given value of \( c_t \), a confidence interval for \( \beta \) at the 1 − \( \alpha \) confidence level is given by 
\[
[\hat{\beta}(c_t, \alpha), \overline{\beta}(c_t, \alpha)],
\]
where 
\[
\overline{\beta}(c_t) = \hat{\beta}(c_t) - z_{1-\alpha}/\sqrt{\hat{\omega}_{112}} \left( \sum_{t=1}^{T} \mathbf{f}_{t-1}^2 - 1 \right)^{-1},
\]
and 
\[
\hat{\beta}(c_t) = \hat{\beta}(c_t) + z_{1-\alpha}/\sqrt{\hat{\omega}_{112}} \left( \sum_{t=1}^{T} \mathbf{f}_{t-1}^2 - 1 \right)^{-1},
\]
and \( z_{1-\alpha} \) denotes the 1 − \( \alpha/2 \) quantile of the standard normal distribution. Here, \( \hat{\beta}(c_t) \) is explicitly written as a function of \( c_t \) to facilitate the discussion below when \( c_t \) is unknown. The estimate of \( \omega_{112} \) is simply given by 
\[
\omega_{112} = \omega_{111} - \omega_{12}^2/\omega_{22}.
\]

In order to get around the issue of an unknown \( c_t \) parameter, we rely on similar methods to those used for the cointegration test. That is, given a confidence interval for \( c_t \), we can calculate \( \beta^*(c_t) \) for all values \( c_t \) in this confidence interval. It is easy to show that \( \beta^*(c_t) \) will be a monotone function of \( c_t \) and it is sufficient to calculate the values at the endpoints of the interval. In fact, if the long-run correlation between \( v_t \) and \( u_t \) is negative (positive), then \( \beta^*(c_t) \) will be decreasing (increasing) in \( c_t \). Suppose the confidence level of the lower bound of \( c_t \) is such that \( Pr(c_t < c_t = 2) \) and that of the upper bound such that \( Pr(c_t > c_t = T) \), with \( x_1 = 2T + 1 \). If \( \omega_{12} < 0 \), a robust confidence interval for \( \beta \), with a confidence level of at least 1 − \( \alpha = 1 - x_1 - x_2 \) is then given by

\[
\text{CI}_J(x) = \left[ \hat{\beta}(c_t(x), x_1), \overline{\beta}(c_t(x), x_2) \right]
\]
and if \( \omega_{12} > 0 \) by

\[
\text{CI}_J(x) = \left[ \hat{\beta}(c_t(x), x_1), \overline{\beta}(c_t(x), x_2) \right].
\]

Thus, if \( x_1 = 0.025 \) and \( x_2 = 0.05 \), a 90% confidence interval for \( \beta \) is obtained. As in the case of the cointegration test in the previous section, the actual confidence level may be higher, and for a given \( x_2, x_1 \) and \( x_2 \), we can be allowed to achieve a desired actual confidence level. Campbell and Yogo (2006) discuss in detail the methods for adjusting \( x_2 \) and \( x_1 \). In fact, as shown by Hjalmarsson (2007), the methods in Campbell and Yogo (2006) can be interpreted as a special case of the FM-OLS framework adapted to AR(p) processes, with the same asymptotic properties under this more restrictive assumption. We can therefore rely on their values for \( x_1 \) and \( x_2 \) as given in Table 2 of Campbell and Yogo (2006). Setting \( x_2 = 0.10 \), and using these values of \( x_1 \) and \( x_2 \), result in a confidence interval for \( \beta \) with confidence level of 90%. For completeness, in the empirical section we also show the plain non-size-adjusted 90% confidence interval that is obtained by setting \( x_1 = 0.025 \) and \( x_2 = 0.05 \).

In terms of practical implementation, we obtain a confidence interval for \( c_t \) by inverting the ADF–GLS unit-root test statistic, and use the Newey–West estimator to calculate all long-run variances and covariances.

### 3. Empirical results

#### 3.1. The data and their univariate properties

We use monthly data on short and long nominal interest rates in Australia, Canada, Hungary, India, Japan, Mexico, New Zealand, Poland, Singapore, South Africa, Sweden, Switzerland, the United Kingdom and the United States. For all countries, the long interest rate is the yield on a benchmark government bond and the short interest rate is a representative three-month rate. Details of the data for each country are given in the appendix. Sample starting dates vary from 1955 to 1997, with the sample for the emerging economies generally shorter owing to poorer market functioning and data keeping. Country-specific samples are shown in Table 1.

We begin by documenting the univariate properties of the data series, applying the ADF–GLS test to all variables. Results are shown in Table 1. The table also shows median unbiased estimates of \( c_t \) and \( c_t^* \), and their 90% confidence intervals which are computed by inverting the ADF–GLS test following the methodology suggested by Stock (1991). The estimate of \( c_t^* \) is used in the Bonferroni EG test if it is smaller than zero.

Two important features stand out in Table 1: all of the interest rate series are highly persistent, but the pure unit-root assumption is likely to be too restrictive. Unbiased estimates of \( c_t \) are in most cases negative, suggesting mean reversion in the data. Moreover, the null hypothesis of a unit root is rejected at the 5% level for the three-month interest rate in Canada, New Zealand, Singapore and Switzerland. The benefits of the Bonferroni EG test are evident in these cases. Most researchers would – rightly so – be reluctant to use traditional cointegration analysis after having detected evidence of stationarity. The Bonferroni EG test, on the other hand, provides a tool for valid inference also in the near-integrated case.

#### 3.2. Cointegration

Having investigated the univariate time-series properties of the data, we now turn to the issue of cointegration between long and short interest rates. As described in Section 2.2.1, cointegration is tested by calculating the EG test statistic for the residuals of the following regression:

\[
i_{jt} = \alpha_j + \beta_j v_{jt-1} + u_{jt},
\]

where \( i_{jt} \) and \( v_{jt} \) are the long and short nominal interest rates, respectively, for each country \( j \). The 5% critical value for the traditional EG test, based on a pure unit-root assumption, is −3.341. For the Bonferroni EG test, however, the critical value depends on \( c_t \) and is given for each country in Table 2.

Among the developed countries, the null of no cointegration is rejected for Australia, Canada, New Zealand, Sweden, Switzerland and the United States at the 5% level. The null can similarly be rejected for Hungary, Mexico, Poland and Singapore. These countries thus satisfy the first of the two necessary conditions for the expectations hypothesis to be a valid description of their term structures. The null of no cointegration cannot be rejected for Japan and the United Kingdom, nor India and South Africa. For Japan, this finding is not surprising given the complex zero-nominal-bound problem that disrupted the Japanese economy for much of the sample. It is more surprising that cointegration does not hold for the United Kingdom. Nonetheless, the finding is in line with evidence from previous studies, such as Cuthbertson (1996). The failure to detect a cointegrating relationship between long and short rates in the United Kingdom may owe to the substantial and long-lived decline in long rates midway through the sample. Following the shift to inflation targeting in the mid-1990s, long-run inflation

---

6 The relevant “endogeneity” parameter that determines the choice of \( x_1 \) and \( x_2 \) (\( \delta \) in Table 2 of Campbell and Yogo, 2006) is now given by the long-run correlation \( \omega_{112}/\sqrt{\omega_{111}\omega_{22}} \).
Given the relative immaturity of financial markets in emerging economies, it is noteworthy that the null hypothesis of no cointegration is rejected in four of our six emerging economies. Only India and South Africa fail the test, with structural changes likely to be the case in both cases. In 1998, the Reserve Bank of India adopted a multiple-intervention approach, a departure from earlier monetary policy that appeared to foster a permanent reduction in long and short interest rates. The relatively long sample available for South Africa clearly makes structural breaks a potential problem. Following the end of apartheid in the early 1990s, international interest in South Africa resumed, accompanied by strong inflows of foreign financial capital, and like several other countries, South Africa adopted inflation targeting in early 2000, with a concomitant reduction in long interest rates.

An important feature of Table 2 is that the critical values of the Bonferroni EG test are to the left of the standard unit-root EG critical value, once uncertainty regarding the exact value of the largest autoregressive root in the data has been taken into account. This

| Table 2 |
| Results from cointegration tests. |
| | EG | Critical value for Bonferroni EG test | Sample |
| Canada | −4.789 | −3.375 | 1956M01–2006M10 |
| Hungary | −4.991 | −3.346 | 1997M03–2004M05 |
| India | −2.493 | −3.341 | 1995M01–2006M10 |
| Japan | −2.220 | −3.341 | 1997M01–2006M10 |
| Mexico | −5.757 | −3.374 | 1995M02–2006M10 |
| Poland | −5.275 | −3.341 | 1994M03–2004M05 |
| South Africa | −2.374 | −3.516 | 1976M01–2004M05 |
| Switzerland | −4.186 | −3.341 | 1975M10–2006M10 |
| United Kingdom | −2.913 | −3.341 | 1978M01–2006M10 |
| United States | −4.034 | −3.364 | 1955M03–2006M10 |

Notes: Critical values are for a test with a nominal size of 5%. The 5% critical value of the standard EG test under a unit-root assumption is equal to −3.341.

| Table 1 |
| Results from unit-root tests. |
| | ADF−GLS | ε | 90% Confidence interval for ε | Sample |
| Australia | −1.841 | −6.231 | [−13.211, 0.426] | 1970M01–2006M10 |
| Canada | −0.952 | −1.261 | [−5.748, 3.001] | 1999M01–2006M10 |
| Hungary | −0.576 | −0.178 | [−3.378, 3.426] | 1976M01–2006M10 |
| Hungary | −0.582 | 0.190 | [−3.762, 3.419] | 1997M01–2006M10 |
| India | −0.403 | 0.171 | [−3.058, 3.567] | 1995M01–2006M10 |
| India | 0.279 | 0.974 | [−1.293, 1.942] | 1995M02–2004M05 |
| Japan | −0.296 | 0.342 | [−2.708, 3.630] | 1995M02–2004M05 |
| Mexico | −0.506 | −0.030 | [−3.479, 3.505] | 1995M02–2004M05 |
| Mexico | −0.938 | −1.231 | [−5.671, 3.015] | 1974M01–2006M10 |
| Australia | 0.759 | 1.329 | [−0.888, 4.046] | 1994M03–2004M0 |
| Australia | 0.516 | 1.163 | [−0.556, 4.138] | 1997M01–2006M10 |
| Sweden | −0.227 | 0.438 | [−2.479, 3.672] | 1997M01–2006M10 |
| SE | −0.877 | −1.018 | [−5.263, 3.082] | 1997M01–2006M10 |
| SE | 0.041 | 0.744 | [−1.793, 3.189] | 1997M01–2006M10 |
| SW | −2.059 | −7.901 | [−15.518, −0.634] | 1997M01–2006M10 |
| UK | −0.480 | 0.023 | [−3.366, 3.521] | 1997M01–2006M10 |
| US | 0.097 | 0.799 | [−1.670, 3.848] | 1997M01–2006M10 |
| US | −0.841 | −0.895 | [−5.069, 3.122] | 1997M01–2006M10 |

Significance at the 5% significance level.

Table 1: Results from unit-root tests.

Table 2: Results from cointegration tests.
adjustment of critical values is crucial, as it is key to valid inference when variables do not have exact unit roots. Failing to make this adjustment tends to make traditional tests of cointegration oversized. As seen here, the standard EG cointegration test, based on the unit-root assumption, and the robust Bonferroni test lead to similar conclusions. However, the Bonferroni test, which is correctly sized under more general conditions than the standard test, gives us greater confidence in rejections of the null hypothesis of no cointegration. That is, by performing inference with the Bonferroni test, one no longer needs to condition on the auxiliary assumption that the data were generated by a pure unit-root process when interpreting the cointegration results. The evidence in favour of cointegration is valid also if the data were generated by a weakly mean-reverting process.

3.3. The cointegrating vector

Having found evidence for cointegration in several countries, we turn to testing whether the cointegrating vector matches the theoretically-suggested value (1, −1). As mentioned in the introduction, researchers have typically overlooked interpretation of the estimated vector, either because of a lack of suitable econometric tools in the case of earlier research, or because it has not been the focus. With the methods developed in this paper, we are able to conduct valid inference about the parameter values. Our focus is the 90% confidence interval for \( \beta \); if this does not cover unity, the null hypothesis of \( \beta = 1 \) is rejected, which is equivalent to rejecting the cointegrating vector (1, −1). Point estimates of \( \beta \) are shown in Table 3, both \( \hat{\beta}_{OLS} \) based on OLS estimation and \( \hat{\beta}_{FM-OLS} \) based on standard unit-root FM-OLS. Confidence intervals are also shown. As discussed in Section 2.2.2, two Bonferroni confidence intervals are presented: (i) the size-adjusted interval and (ii) the non-size-adjusted interval. Both methods lead to qualitatively similar results, and it is evident that the size adjustment is not crucial in this particular application.

For all 10 countries in which cointegration was detected using the Bonferroni EG test, the cointegrating vector differs significantly from the values predicted by theory. That is, despite the presence of cointegration, the expectations hypothesis is rejected as a description of the term structure. Interestingly, the values are similar across countries, with \( \beta \) significantly smaller than one and clustered between 0.4 and 0.8. The confidence intervals also overlap substantially. The finding of a slope coefficient less than one is not unique to this paper; for example, Engle and Granger (1987), Boothe (1991) and MacDonald and Speight (1991) report similar coefficients using data from different countries and sample periods. Unlike previous studies, however, we have the tools to reliably conclude that the deviations from the theoretically-implied relationship are statistically significant.

3.4. Interpretation and discussion

The joint finding of cointegration with a slope coefficient less than one is intriguing, as it suggests that the term structures of many countries are driven not just by expectations of future short interest rates and stationary term premia but by an additional near-integrated component that covaries systematically with the short interest rate. Rather than an outright rejection of the expectations hypothesis, it suggests that something more is at work. One candidate for this additional component is a time-varying but persistent risk premium linked to the state of the term structure. Campbell and Shiller (1991) propose exactly this to explain why coefficients in their yield-spread regressions deviate from theory. Specifically, they suggest time-varying risk premia as a rationale for why “long rates underreact to short-term interest rates” (p. 513). Dai and Singleton (2002) go further, showing that yield-spread regressions using risk-premium-adjusted yields from a broad class of affine term structure models recover expectations hypothesis predicted coefficients. To match the empirical findings of Fama and Bliss (1987) and Campbell and Shiller (1991), Dai and Singleton’s model-estimated risk premia are negatively correlated with the short interest rate.

To see how this explanation fits into the present application, consider a generalisation of Eq. (2) that retains the basic structure of the expectations hypothesis but generalises the term premium:

\[
\begin{align*}
\hat{\beta}_t - \hat{\beta}^1 &= \frac{1}{n} \sum_{t=1}^{n} \sum_{j=1}^{p} E_t(\Delta X_{t+j}) + \frac{\hat{\beta}}{C_0}. \\
\end{align*}
\]

(15)

The new term premium \( \hat{\beta}_t \), is the sum of two components, the stationary component posited earlier, \( \hat{\beta}^1 \), plus a new component, \( \frac{\hat{\beta}}{C_0} \), that is near-integrated and covaries with the short interest rate. Specifically, assume that \( \frac{\hat{\beta}}{C_0} = \tilde{\chi}_t \) such that if \( \hat{\beta}_t \) is near integrated, so too is \( \frac{\hat{\beta}}{C_0} \) and they will in fact be cointegrated. Moving this term to the left-hand side,

\[
\begin{align*}
\hat{\beta}_t - \hat{\beta}^1 - \frac{\hat{\beta}}{C_0} &= \hat{\beta}_t - (1 + \hat{\beta}^1) = \frac{1}{n} \sum_{t=1}^{n} \sum_{j=1}^{p} E_t(\Delta X_{t+j}) + \hat{\beta}_t, \\
\end{align*}
\]

(16)

our empirical finding of a slope coefficient of less than unity leads us to infer that \( \hat{\beta}^1 \) is also negative. That is, the near-integrated element of \( \frac{\hat{\beta}}{C_0} \) appears to covary inversely with the short interest rate, consistent with the existing literature on the term structure for the United States described above. While the empirical analysis in this paper has by no means identified such risk premia, it is a

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Estimates of the cointegrating vector.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>90% Confidence interval for ( \beta ) (i)</td>
</tr>
<tr>
<td>Australia</td>
<td>[0.600, 0.728]</td>
</tr>
<tr>
<td>Canada</td>
<td>[0.654, 0.763]</td>
</tr>
<tr>
<td>Hungary</td>
<td>[0.739, 0.857]</td>
</tr>
<tr>
<td>India</td>
<td>–</td>
</tr>
<tr>
<td>Japan</td>
<td>–</td>
</tr>
<tr>
<td>Mexico</td>
<td>[0.753, 0.852]</td>
</tr>
<tr>
<td>New Zealand</td>
<td>[0.578, 0.686]</td>
</tr>
<tr>
<td>Poland</td>
<td>[0.747, 0.825]</td>
</tr>
<tr>
<td>Singapore</td>
<td>[0.546, 0.776]</td>
</tr>
<tr>
<td>South Africa</td>
<td>–</td>
</tr>
<tr>
<td>Sweden</td>
<td>[0.629, 0.773]</td>
</tr>
<tr>
<td>Switzerland</td>
<td>[0.343, 0.446]</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>–</td>
</tr>
<tr>
<td>United States</td>
<td>[0.772, 0.911]</td>
</tr>
</tbody>
</table>

Notes: The 90% confidence interval (i) has been size adjusted, whereas (ii) has not.
plausible explanation for the results in Table 3, as it is likely to be a common characteristic of the behaviour of investors across countries and markets. Naturally, other explanations for the failure of the expectations hypothesis abound, such as departures from rational expectations or sluggish adjustment of expectations about persistent changes in short rates. For some countries in our sample, gradual adjustment of long-run inflation expectations during the shift to a lower-inflation environment may have delayed adjustment of long interest rates, as suggested by Kozicki and Tinsley (2005) for the United States during the 1980s. However, given the similar results we find across countries and time periods, an explanation based on the failure of expectations to adjust quickly or rationally seems less likely to be the common explanation.

4. Conclusion

Nominal interest rates are likely to be stationary, albeit slowly mean reverting. The theoretical determinants of interest rates, such as agents’ risk aversion and long-run growth rates of the macroeconomy, imply that real interest rates possess a long-run equilibrium value. Historically, nominal interest rates have fluctuated for many centuries within a fairly narrow band that encompasses values common today. However, their typically slow rate of mean reversion poses a problem for traditional unit-root tests, and confronted with series that behave as integrated in finite samples, many researchers have proceeded to apply cointegration methods based on the unit-root assumption. Unfortunately, empirical analysis based on this assumption can be misleading. Even small deviations from the unit-root assumption can lead to large size distortions and thus mistaken inference.

This paper illustrates the application of near-integration methods of cointegration by testing the expectations hypothesis on term-structure data from numerous countries. The econometric framework is robust to deviations from the unit-root hypothesis by nesting the high-persistence and unit-root cases. The empirical results strongly support cointegration between long and short interest rates in several developed and emerging economies, a necessary condition for the expectations hypothesis to hold. However, for all 10 countries in which cointegration is present, the theoretically-predicted vector (1, –1) is rejected. Moreover, the estimated vectors are similar across countries, with long rates moving by less than short rates, on average. While the causes for this are far from obvious, our empirical findings are consistent with the presence of time-varying bond term premia that are themselves highly persistent and co-vary inversely with the short rate. Such reasoning is in a similar spirit to explanations put forward for the failure of the expectations hypothesis in the excess-returns literature, and with the stylised fact that risk premia are countercyclical. Our empirical analysis is not sufficiently structural to test this hypothesis, and looking deeper requires reliable estimates of risk premia, a complex issue in itself. We nevertheless believe it to be an interesting direction for future research.

Appendix

This section details the data types and sources used in the analysis. For Australia, Canada, India, Japan, New Zealand, Sweden, Switzerland, the United Kingdom and the United States, the long interest rate is the yield on the benchmark 10-year government bond, for Hungary, Poland and Singapore, the yield on the five-year government bond, for Mexico, the yield on the three-year government bond and, finally, for South Africa, the yield on the 20-year government bond. The long interest rates are measured as yields on coupon securities. Turning to short interest rates, these are the three-month bank bill rate for Australia and New Zealand, the three-month corporate paper rate for Canada, the three-month certificate of deposit rate for India and Japan, the three-month interbank rate for Switzerland and the United Kingdom, and the three-month treasury bill rate for Hungary, Mexico, Poland, Singapore, South Africa, Sweden and the United States. The data were sourced from the Board of Governors of the Federal Reserve System, the International Monetary Fund, and Global Financial Data.

Following Campbell and Shiller (1987), the long interest rates in our sample are the yields on specific coupon securities, as synthetic, constant-maturity, zero-coupon yields are not readily available for all countries. As pointed out by Shea (1992), conducting our analysis using yields from coupon securities relies on approximations that could distort inference. For the United Kingdom and the United States, we are able to check whether our choice materially affects our conclusions by performing our analysis on synthetic constant-maturity yields provided by the respective central banks. The results of this exercise (reported below) indicate that our findings are not sensitive to the choice of how the long interest rate is measured. Denote the 10-year zero-coupon yield as \( z_{10, UK} \) and \( z_{10, US} \) for the United Kingdom and United States respectively as the long interest rate. The sample period for the United Kingdom is the same as for the main data. For the United States, the sample

<table>
<thead>
<tr>
<th>( b )</th>
<th>( ADF–GLS )</th>
<th>( \hat{c} )</th>
<th>90% Confidence interval for ( c )</th>
<th>Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_{UK} )</td>
<td>-0.880</td>
<td>-1.030</td>
<td>[–5.289, 3.079]</td>
<td>1978M01–2006M10</td>
</tr>
<tr>
<td>( \hat{b}_{UK} )</td>
<td>0.349</td>
<td>1.034</td>
<td>[–1.162, 3.978]</td>
<td>1978M01–2006M10</td>
</tr>
<tr>
<td>( \hat{b}_{UK} )</td>
<td>-0.880</td>
<td>-1.030</td>
<td>[–5.289, 3.079]</td>
<td>1978M01–2006M10</td>
</tr>
<tr>
<td>( \hat{b}_{UK} )</td>
<td>0.097</td>
<td>0.799</td>
<td>[–1.670, 3.848]</td>
<td>1978M01–2006M10</td>
</tr>
<tr>
<td>( b_{US} )</td>
<td>-1.176</td>
<td>-2.225</td>
<td>[–7.269, 2.541]</td>
<td>1973M01–2006M10</td>
</tr>
<tr>
<td>( \hat{b}_{US} )</td>
<td>-1.176</td>
<td>-2.225</td>
<td>[–7.269, 2.541]</td>
<td>1973M01–2006M10</td>
</tr>
<tr>
<td>( \hat{b}_{US} )</td>
<td>-1.121</td>
<td>-2.387</td>
<td>[–7.520, 2.472]</td>
<td>1973M01–2006M10</td>
</tr>
</tbody>
</table>

Notes: * indicates significance at the 5% significance level.
period is considerably shorter than originally, namely January 1973–October 2006; in order to be able to make direct comparisons for the US data, we therefore also redo the analysis using the coupon-bearing bond rate over this shorter sample.

Table A.1 shows the results from unit-root tests and provides estimates of the local-to-unity parameters for the different series. In Table A.2, results from the cointegration tests are presented. As can be seen from the tables, the results are qualitatively the same as when we use the original data – cointegration is supported in the United States but not in the United Kingdom. As we found no evidence of cointegration for the United Kingdom, inference regarding the cointegrating vector can only be carried out for the United States. Results from this exercise are shown in Table A.3. While there are some minor quantitative differences, the data-sets still communicate a message very similar to that seen previously. The cointegrating vector (1, –1) is clearly rejected and estimates suggest a value of around (1, –0.7).

### References


